

Proof reading of an integration problem

Check the following integration steps. Are there any mistakes that lead to the different answers in Method 1 and Method 2?

Method 1

Evaluate $\int \frac{x^2}{\sqrt{4-x^2}} dx$.

We have the standard integral: $\int \frac{1}{\sqrt{4-x^2}} dx = \sin^{-1}\left(\frac{x}{2}\right) + c$.

$$\begin{aligned} I &= \int \frac{x^2}{\sqrt{4-x^2}} dx = \int \frac{4-(4-x^2)}{\sqrt{4-x^2}} dx = 4 \int \frac{1}{\sqrt{4-x^2}} dx - \int \sqrt{4-x^2} dx \\ &= 4\sin^{-1}\left(\frac{x}{2}\right) - [x\sqrt{4-x^2} - \int x d(\sqrt{4-x^2})] , \text{ integration by parts.} \\ &= 4\sin^{-1}\left(\frac{x}{2}\right) - x\sqrt{4-x^2} + \int x \frac{-2x}{2\sqrt{4-x^2}} dx = 4\sin^{-1}\left(\frac{x}{2}\right) - x\sqrt{4-x^2} - I \\ \therefore 2I &= 4\sin^{-1}\left(\frac{x}{2}\right) - x\sqrt{4-x^2} \\ \therefore I &= 2\sin^{-1}\left(\frac{x}{2}\right) - \frac{1}{2}x\sqrt{4-x^2} + C \quad \dots (1) \end{aligned}$$

Method 2

$$I = \int \frac{x^2}{\sqrt{4-x^2}} dx$$

Let $u^2 = 4 - x^2$, $2u du = -2x dx$, $u du = -x dx$

$$I = - \int \frac{\sqrt{4-u^2}}{u} u du = - \int \sqrt{4-u^2} du$$

Let $u = 2 \sin t$, $du = 2 \cos t dt$

$$I = - \int 2 \cos t (2 \cos t dt) = -4 \int \cos^2 t dt = -4 \int \left(\frac{1+\cos 2t}{2}\right) dt = -2[t + \sin 2t] + c$$

$$= -2t - 2 \sin t \cos t + c = -2\sin^{-1}\left(\frac{u}{2}\right) - u\sqrt{1 - \left(\frac{u}{2}\right)^2} + c = -2\sin^{-1}\left(\frac{u}{2}\right) - \frac{1}{2}u\sqrt{u^2 - 1} + c$$

$$= -2\sin^{-1}\left(\frac{\sqrt{4-x^2}}{2}\right) - \frac{1}{2}x\sqrt{4-x^2} + c \quad \dots (2)$$